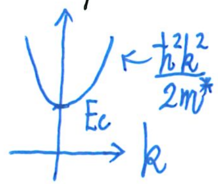
E. Conduction Electrons in semiconductors with isotropic band

$$\mathcal{E}(\vec{k}) = E_c + \frac{\hbar^2 k^2}{2m^*}$$



retains $\tau(k)$ or $\tau(\mathcal{E})$ [no long $\tau(\vec{k})$]

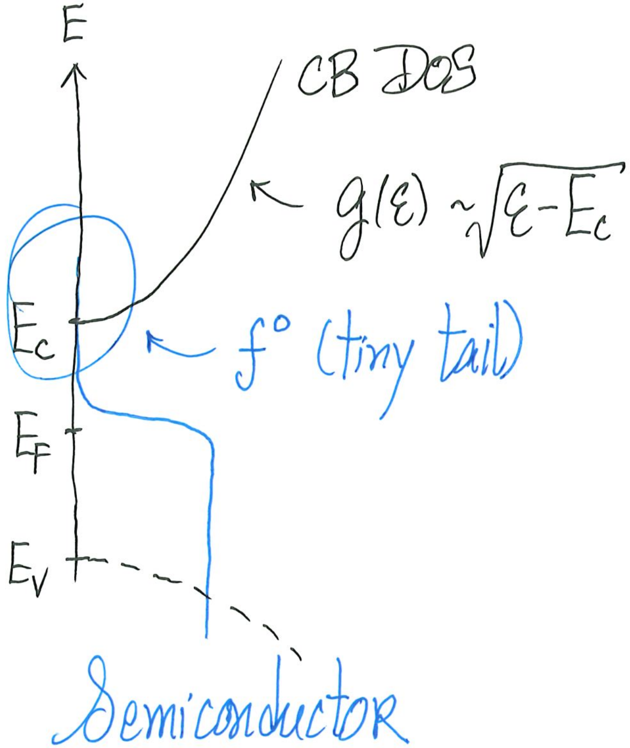
$$\sigma_{xx} = \sigma = \frac{e^2}{V} \sum_{\vec{k}} \sum_{spin} \left(-\frac{\partial f^0}{\partial \mathcal{E}}\right) v_x^2(\vec{k})\, \tau(\vec{k}) = \frac{e^2}{3V} \sum_{\vec{k}} \sum_{spin} \left(-\frac{\partial f^0}{\partial \mathcal{E}}\right) v^2(k)\, \tau(k)$$

$$= \frac{e^2}{3} \int d^3k \, \frac{2}{(2\pi)^3} \left(-\frac{\partial f^0}{\partial \mathcal{E}}\right) v^2(k)\, \tau(k)$$

$$= \frac{e^2}{3} \int_{E_c}^{\text{Top of CB}} \underbrace{\frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} \sqrt{\mathcal{E} - E_c}}_{g(\mathcal{E})} \left(-\frac{\partial f^0}{\partial \mathcal{E}}\right) v^2 \tau \, d\mathcal{E} \qquad (33)$$

Semiconductors: electrons in CB form a non-degenerate gas

$$n = \underbrace{N_c}_{\text{effective \# states (at } E_F)} \underbrace{e^{-(E_c - E_F)/kT}}_{\text{only occupied with tiny probability}}$$



$$\overset{\text{for } \varepsilon \text{ in CB}}{f^0(\varepsilon)} \approx e^{-(\varepsilon - E_F)/kT} \quad \text{(classical/Boltzmann statistics)}$$

$$\left(1 - f^0(\varepsilon)\right) \approx 1$$

$$\therefore (a) \left(-\frac{\partial f^0}{\partial \varepsilon}\right) = \frac{1}{kT} f^0 \underbrace{(1 - f^0)}_{\approx 1} \approx \frac{1}{kT} \underbrace{e^{-(\varepsilon - E_F)/kT}}_{\text{tiny in CB}}$$

$$\text{so } (b) \int_{E_c}^{\text{Top of CB}} d\varepsilon \rightarrow \int_{E_c}^{\infty} d\varepsilon \quad \text{in Eq. (33) is OK}$$

(Discussions here are not valid for metals and heavily doped semiconductors)

n = electron number density in CB

$$= \int_{E_c}^{\infty} \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon - E_c}\cdot \underbrace{e^{-(\varepsilon-E_F)/kT}}_{f^0}\, d\varepsilon \qquad (34)$$

For such a gas, $\langle E\rangle = \frac{3}{2}N_e kT \Rightarrow \frac{\langle E\rangle}{V} = \frac{3}{2}nkT$ (statistical physics)[+]

$$\Rightarrow n = \frac{2}{3}\frac{1}{kT}\left(\frac{\langle E\rangle}{V}\right)$$

$$\therefore\ n = \frac{2}{3}\frac{1}{kT}\int_{E_c}^{\infty}\frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)(\varepsilon - E_c)^{3/2} e^{-(\varepsilon-E_F)/kT}\, d\varepsilon$$

[+] This is $U = \frac{3}{2}NkT$ or $\langle E\rangle = \frac{3}{2}NkT$ in statistical physics.

$$\sigma = \frac{e^2}{3}\int_{E_c}^{\infty} \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon - E_c}\,\left(\frac{1}{kT}e^{-(\varepsilon-E_F)/kT}\right)\overbrace{\left(\frac{2}{m^*}(\varepsilon-E_c)\right)}^{v^2(\varepsilon)}\tau(\varepsilon)\,d\varepsilon$$

$(\varepsilon - E_c) = \frac{1}{2}m^* v^2(\varepsilon)$

$$= \frac{2e^2}{3}\frac{1}{m^*}\frac{1}{kT}\int_{E_c}^{\infty}\frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}(\varepsilon-E_c)^{3/2}\,\tau(\varepsilon)\,e^{-(\varepsilon-E_F)/kT}\,d\varepsilon$$

times $\left(\frac{n}{n}\right)$

$$= \frac{ne^2}{m^*}\,\frac{\cancel{\frac{2}{3}\frac{1}{kT}}\int_{E_c}^{\infty}\cancel{\frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}}(\varepsilon-E_c)^{3/2}\,\tau(\varepsilon)\,e^{-(\varepsilon-E_F)/kT}\,d\varepsilon}{\cancel{\frac{2}{3}\frac{1}{kT}}\int_{E_c}^{\infty}\cancel{\frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}}(\varepsilon-E_c)^{3/2}\,e^{-(\varepsilon-E_F)/kT}\,d\varepsilon}$$

electron # density in CB

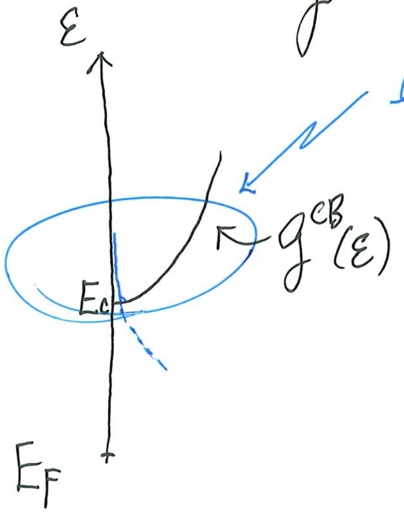
$$= \frac{ne^2}{m^*}\,\overline{\tau} \qquad (35)$$

(a familiar result with non-trivial meaning!)

$$\boxed{\overline{\tau} \equiv \frac{\int_{E_c}^{\infty}\tau(\varepsilon)(\varepsilon-E_c)^{3/2}e^{-(\varepsilon-E_F)/kT}\,d\varepsilon}{\int_{E_c}^{\infty}(\varepsilon-E_c)^{3/2}e^{-(\varepsilon-E_F)/kT}\,d\varepsilon}} \qquad (36)$$

(Key result) (Semiconductors)

Meaning



electrons in CB have many empty states to scatter into
$\Rightarrow$ need to consider $\underbrace{\tau(\varepsilon)}_{\text{for } \varepsilon \text{ in CB}^{+}}$

Use QM, obtain $\tau(\varepsilon)$ for

ε in CB

Then, Transport Theory indicates a $\underbrace{\text{non-trivial average}}_{\text{due to non-equilibrium } f \text{ in steady state}}$ to obtain $\bar{\tau}$ (Eq. (36))

weighting factor $\sim (\varepsilon - E_c)^{3/2} e^{-(\varepsilon - E_F)/kT}$

It also follows that the mobility $\mu = \dfrac{e\bar{\tau}}{m^*}$, also involving the same $\bar{\tau}$.

$^{+}$ In a metal, the electrons at Fermi surface matter, due to Pauli Principle (degenerate gas)

Result Eq. (36) can be applied (in semiconductors) when we encounter a function $F(\tau)$ for averging.

$$\frac{1}{3}\int_{E_c}^{\infty} \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon - E_c}\left(-\frac{\partial f^0}{\partial \varepsilon}\right) v^2 F(\tau)$$

parabolic band assumed

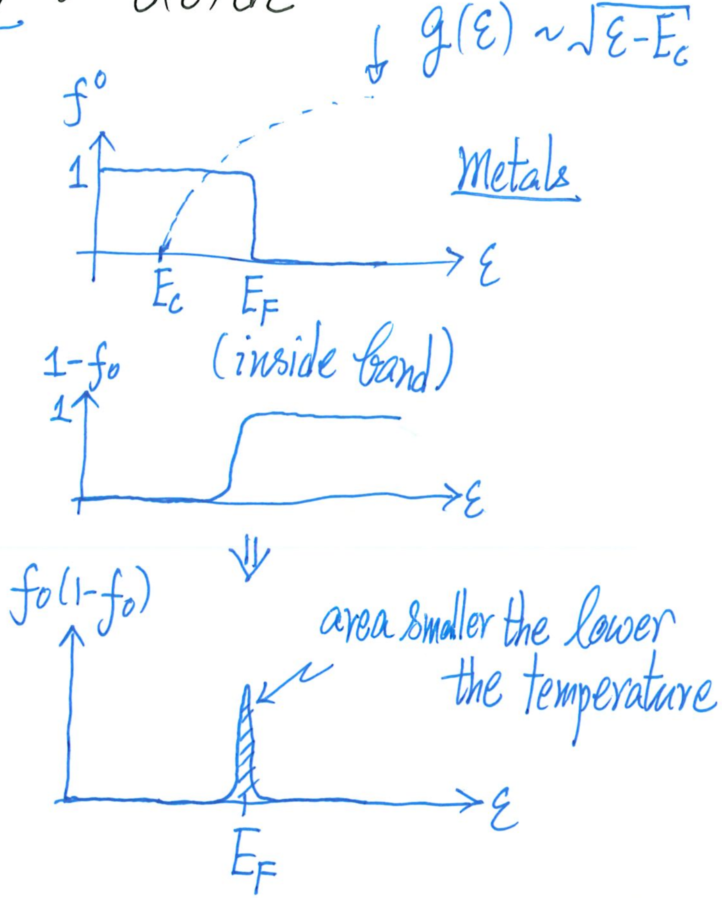
$$= \frac{n}{m^*}\overline{F(\tau)}$$

where
$$\overline{F(\tau)} \equiv \frac{\int_{E_c}^{\infty} F(\tau)(\varepsilon - E_c)^{3/2} e^{-(\varepsilon - E_F)/kT} d\varepsilon}{\int_{E_c}^{\infty}(\varepsilon - E_c)^{3/2} e^{-(\varepsilon - E_F)/kT} d\varepsilon} \qquad (37)$$

Rely on engaging the tail of Fermi function.

Aside: For metals, we go back to Eq. (33)

$$\sigma = \frac{e^2}{3}\int_{E_c}^{\infty} \underbrace{\frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon - E_c}}_{g(\varepsilon)}\ \underbrace{\delta(\varepsilon - E_F)}_{-\left(\frac{\partial f^0}{\partial \varepsilon}\right)}\ v^2 \tau(\varepsilon)\, d\varepsilon$$



$$\because\ -\left(\frac{\partial f^0}{\partial \varepsilon}\right) = \frac{1}{kT} f^0(1-f^0)$$
$$= \delta(\varepsilon - E_F)$$

$$\therefore\ f^0(1-f_0) = kT\,\delta(\varepsilon - E_F)$$

$$\sigma = \frac{e^2}{3}\, g(E_F)\, v_F^2\, \underbrace{\tau(E_F)}_{\substack{\text{picks up } \tau \\ \text{at } \varepsilon = E_F}}$$

accounts for all electrons

But $n = \int_{E_c}^{E_F} g(\varepsilon)\, d\varepsilon$ (ideal Fermi Gas physics)

$$= \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}(E_F - E_c)^{3/2}\cdot\frac{2}{3} = \frac{2}{3}\, g(E_F)\cdot(E_F - E_c)$$

$$E_F - E_c = \frac{\hbar^2 k_F^2}{2m^*} = \frac{1}{2} m^* \underbrace{v_F^2}_{\text{Fermi Velocity}} \Rightarrow n = \frac{1}{3} g(E_F) \cdot m^* v_F^2$$

$$\therefore \sigma = \frac{ne^2}{3} \frac{g(E_F)\, v_F^2\, \tau(E_F)}{\frac{1}{3} g(E_F) \cdot m^* v_F^2} = \underbrace{\frac{ne^2 \tau(E_F)}{m^*}}_{\text{"same" result/form}}$$

but "τ" in $\frac{ne^2\tau}{m^*}$ for metals picks up $\underbrace{\tau(E_F)}$

only need to find τ (or $1/\tau$) from scattering processes for electron energy at E_F (due to Pauli Exclusion)